Analog Electronics
ENEE236

Operational Amplifiers

Instructor
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Operational Amplifiers

- Early Operational Amplifiers were constructed with vacuum tubes and were used in analog computers to perform mathematical operations.
- Even as late as 1965, vacuum tube operational amplifiers were still in use and cost in the range of $75.
- These days, they are linear Integrated circuits (IC) that use low voltage dc supplies , they are reliable and inexpensive
- The operational amplifier has become so cheap in price (often less than $1.00 per unit) and it can be used in so many applications

Early Vacum Tube Operational Amplifiers

The Philbrick Operational Amplifier (1952)

From "Operational Amplifier", by Tony van Roon: http://www.uoguelph.ca/~antoon/gadgets/741/741.html

Operational Amplifiers

Operational was used as a descriptor early-on because this form of amplifier can perform operations of :

- **Adding signals**
- **Subtracting signals**
- **Integrating signals,** $\int x(t)dt$
- **Differentiation of signals,**

The applications of operational amplifiers (shortened to op amp) have grown beyond those listed above.

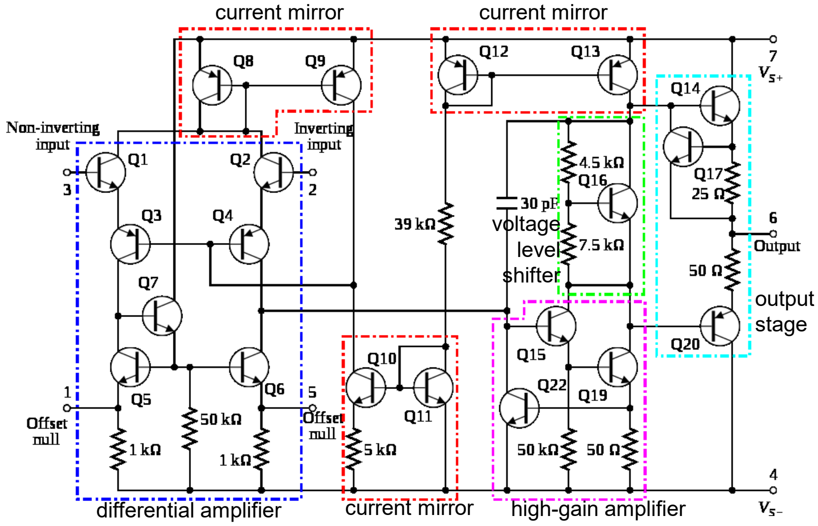
What can you do with Op amps?

- You can make music louder when they are used in stereo equipment.
- You can amplify the heartbeat by using them in medical cardiographs.
- You can use them as comparators in heating systems.
- You can use them for Math operations
- And many other applications in all fields of engineering

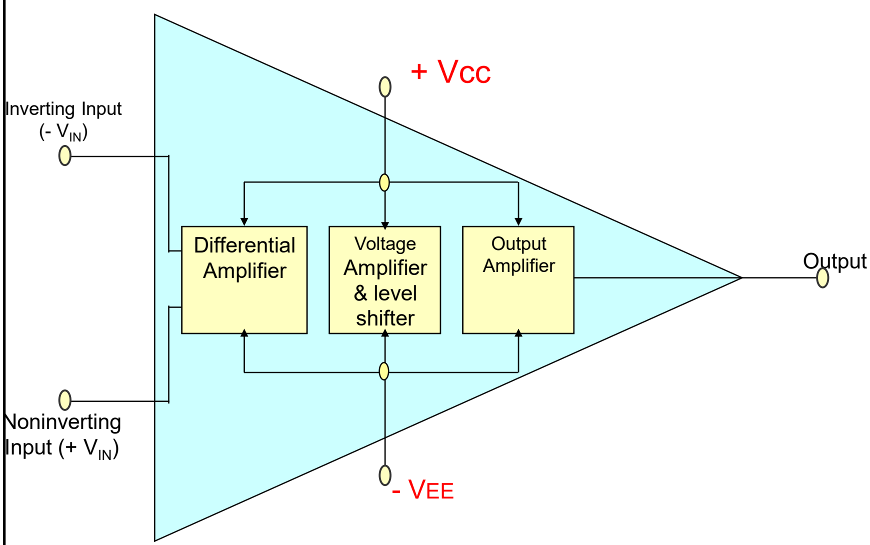
Operational Amplifiers

- In this course we will be concerned with ***how to use the op amp as a device.***
- The internal configuration (design) is beyond the scope of our study and can be covered in an advanced electronics course.
- The complexity is illustrated in the following block diagram and detailed circuit.

741 Op-Amp Schematic



OP-AMP BLOCK DIAGRAM



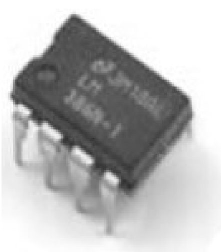
OP-AMP CHARACTERISTICS

1. Very high input impedance (in mega ohms)
2. Very high gain (> 100,000)
3. Very low output impedance (in ohms)

OP-AMP is a differential, voltage amplifier with high gain.

Operational Amplifiers

Fortunately, we do not have to *assemble* a circuit with so many transistors and resistors in order to get and use the op amp The circuit in the previous slide is usually encapsulated into a dual in-line pack (DIP) or any other suitable package type



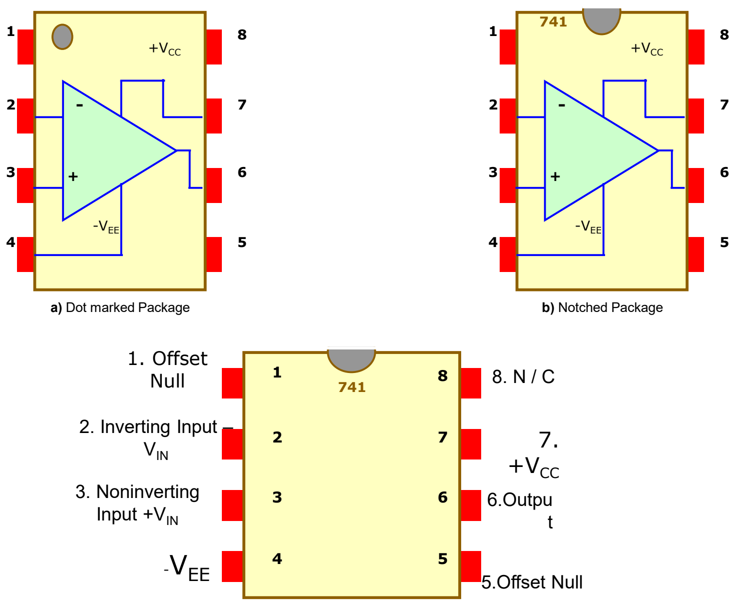
(a) Op Amp 741
8-pins DIP package

(b) OPA547FKTWT
DIP SMT package

(c) TO-5 metal can
8-Leads package

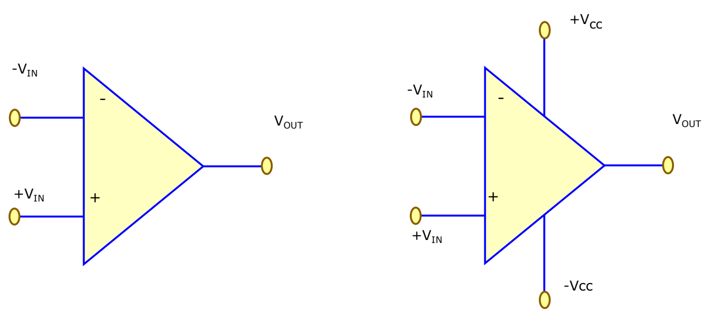
Op Amp packages

OP-AMP pins identification



Op Amp pins Description

Symbol of OP-AMP



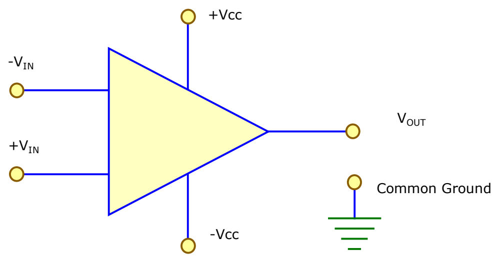
(a) Without power connection **(b)** With power connection

Op Amp Schematic Symbols

Most Op Amps require dual power supply with common ground

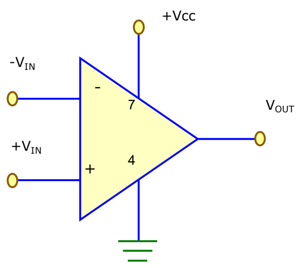
Positive Supply +Vcc to pin7 in 741 opamp

Negative Supply -Vcc to pin4 in 741 opamp

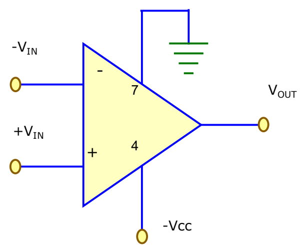


Dual Supply Voltages connection

Some Op Amps work on single supply also (with some restrictions)



(a) Single Positive Voltage

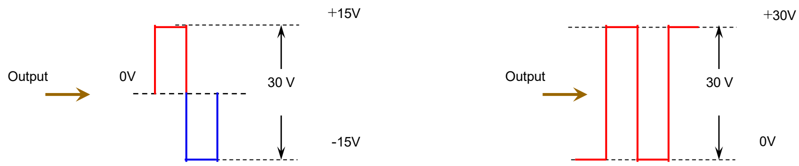


Single Negative Voltage

Single Supply Voltages connection

Advantage of dual power supply

Using dual power supply will let the op amp to output true AC voltage.



Op Amp powered from Dual supply **Op Amp powered from Single supply**

What is dual power supply?

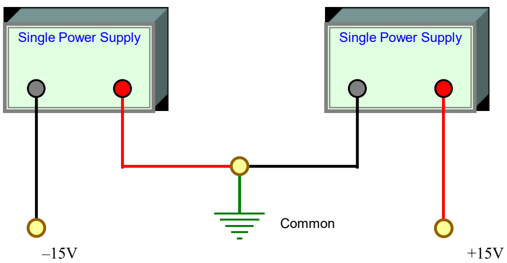
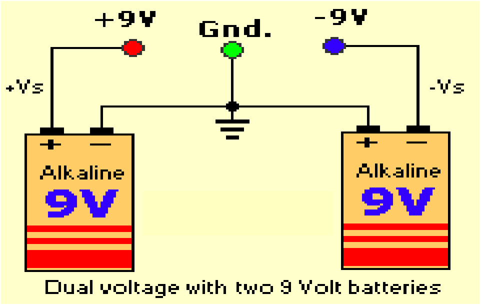


Figure 18 Dual Power Supply

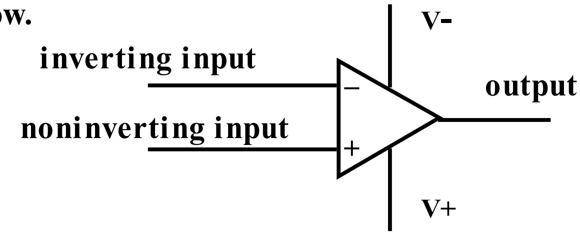
How can you make a dual power supply using two 9V batteries?

What is the voltage between + of first battery and – of second battery?



Operational Amplifiers

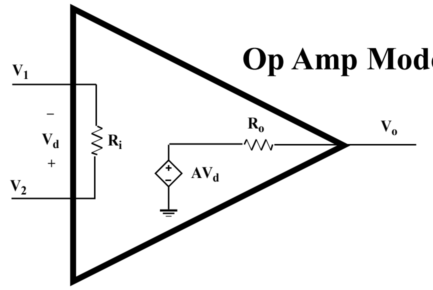
The basic op amp with supply voltage included is shown in the diagram below.



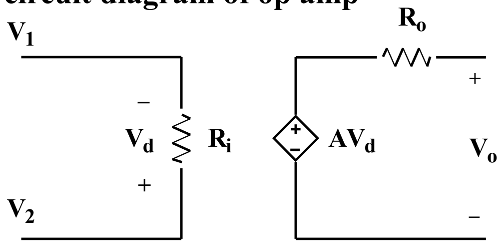
In most cases only the two inputs and the output are shown for the op amp.
However, one should keep in mind that supply voltage is required, and a ground.

Operational Amplifiers Model (Linear Region)

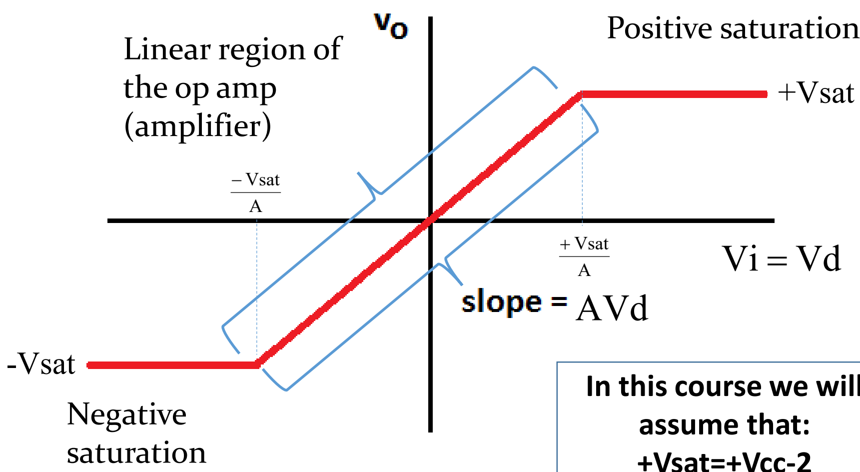
A model of the op amp, with respect to the symbol, is shown below.



Working circuit diagram of op amp

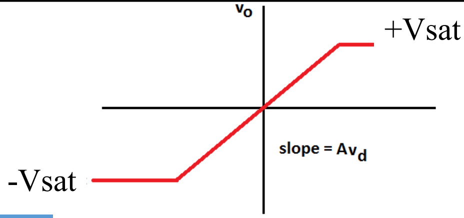


Voltage Transfer Characteristic



In this course we will assume that:
+Vsat=+Vcc-2
-Vsat=-Vcc+2
In reality value must be taken from data sheets

Output Voltage



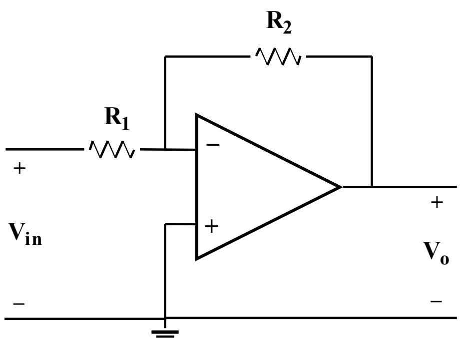
- Real Op Amp

Mode of operation	Voltage Range	Output Voltage
Positive Saturation	**$A\ V_d > +Vsat$**	**$v_o = +Vsat =\sim +Vcc-2$**
Linear Region	**$-Vsat < A\ V_d < +Vsat^+$**	**$v_o = A\ V_d$**
Negative Saturation	**$A\ V_d < -Vsat$**	**$v_o = -Vsat =\sim -Vcc+2$**

- The voltage produced by the dependent voltage source inside the op amp is limited by the voltage applied to the positive and negative rails and +/-Vsat level which is approximated by the formulas above
- Also note that opamps are capable of supplying/sourcing limited amount of current (Io(max)) which can affect its output voltage

Operational Amplifiers Analysis

As an application of the previous model, consider the following configuration.
Find V_o as a function of V_{in} and the resistors R_1 and R_2.



Op amp functional circuit.

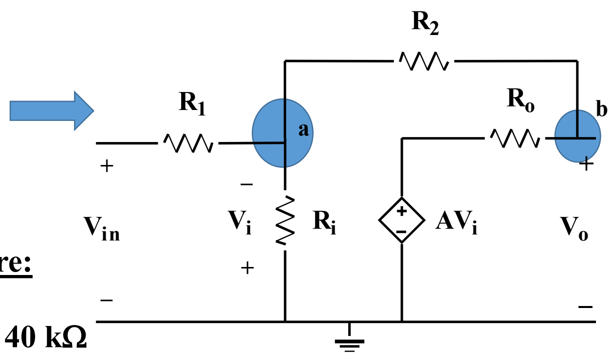
Operational Amplifiers

$\mathrm{Vi} = \mathrm{Vd} = V_p - V_n = V_+ - V_-$ differential input voltage

$V_p = V_+$ voltage at the non-inverting input

$V_n = V_-$ voltage at the inverting input

Equivalent circuit

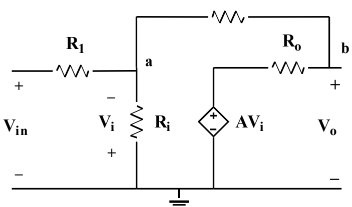


Component values are:

R_1 = 10 kΩ **R_2 = 40 kΩ**
R_o = 50 Ω
A = 100,000 **R_i = 1 meg Ω**

Operational Amplifiers

Exact solution
We can write the following equations for nodes a and b.



R_1 = 10 kΩ **R_2 = 40 kΩ**
R_o = 50 Ω
A = 100,000 **R_i = 1 meg Ω**

KCL at A

$$\frac{V_{in}+V_i}{R_1} = \frac{-V_i}{R_i} - \frac{V_i+V_o}{R_2} \qquad (1)$$

KCL at B

$$V_o = R_o\left[\frac{-(V_i+V_o)}{R_2}\right] + AV_i \qquad (2)$$

Operational Amplifiers

Equation 1 simplifies to;

R_1 = 10 kΩ R$_2$ = 40 kΩ
R_o = 50 Ω
A = 100,000 R_i = 1 meg Ω

$$\frac{V_{in}+V_i}{10k}=\frac{-V_i}{1000k}-\frac{V_i+V_o}{40k}$$

$$-25\mathrm{V_o}-126\mathrm{V_i}=100\mathrm{V_{in}} \qquad (3)$$

Equation 2 simplifies to;

$$\mathrm{V_o}=50\left[\frac{-(V_i+V_o)}{40k}\right]+100{,}000V_i$$

$$\left(4.005.10^5\right)V_o-\left(4.10^9\right)V_i=0 \qquad (4)$$

Operational Amplifiers

From Equations (3) and (4) we find;

$$\boldsymbol{V_o}=-3.99\boldsymbol{V_{in}} \qquad \textbf{(5)}$$

This is an expected answer.

Fortunately, we are not required to do elaborate circuit analysis, as above, to find the relationship between the output and input of an op amp. Simplifying the analysis is our next consideration.

Operational Amplifiers Models

For most operational amplifiers,
R_i is 1 Meg Ω or larger and
R_o is around 50 Ω or less.
The open-loop gain, A, is greater than 100,000.

Ideal Op Amp Model:

The following assumptions are made for the ideal op amp.

$$1.\ Infinite\ \ open-loop\ \ gain; \Rightarrow A \cong \infty$$

$$2.\ Zero\ \ output\ \ ohms; \Rightarrow R_o = 0$$

$$3.\ Infinite\ \ input\ \ ohms; \Rightarrow R_i = \infty$$

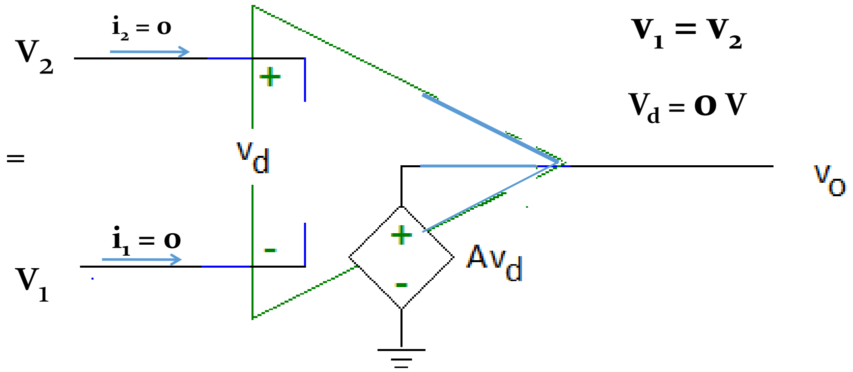
Ideal Op Amp Model

Because Ri is equal to ∞Ω,

the voltage across Ri is 0 V.

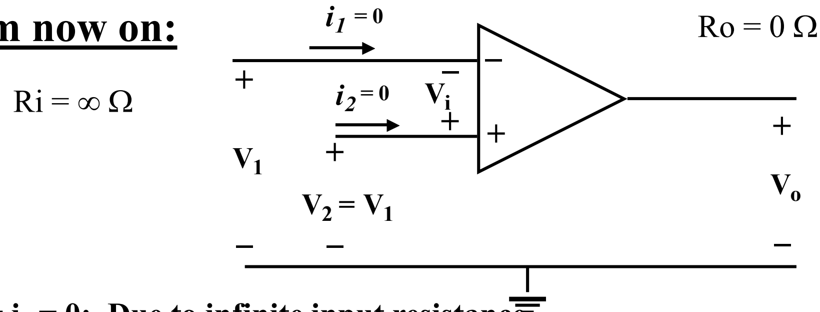
$V_1 = V_2$

$V_d = 0$ V



Important Note: **Ideal Op Amp**

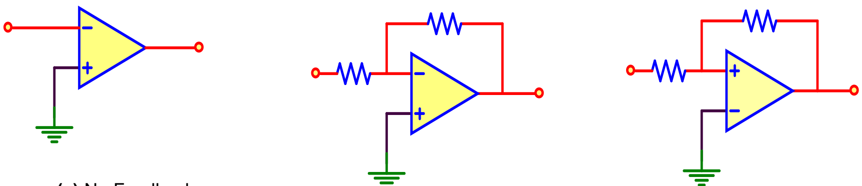
Only Ideal Op Amp Model will be used from now on:



(a) $i_1 = i_2 = 0$: Due to infinite input resistance.
(b) V_i is negligibly small; $V_1 = V_2$.

The op amp forces the voltage at the inverting input terminal to be equal to the voltage at the noninverting input terminal if there is some component connecting the output terminal to the inverting input terminal.

OP-AMP CONFIGURATIONS



(a) No Feedback (open loop comparator circuit)

(b) Negative Feedback

(c) Positive Feedback

- Negative feedback : Feedback to the inverting input (Used in amplifiers)- Linear Application
- Positive feedback : Feedback to the non inverting input (Used in oscillators) and Schmitt triggers (comparators with hysterisis)-non Linear Application
- No feedback : Open loop (used in comparators)- non linear application

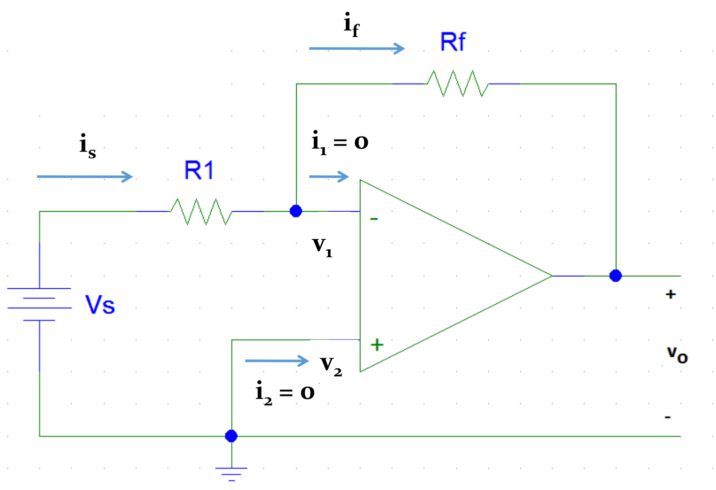
OP-AMPS WITH NEGATIVE FEEDBACK

The two basic amplifier circuits with negative feedback are:

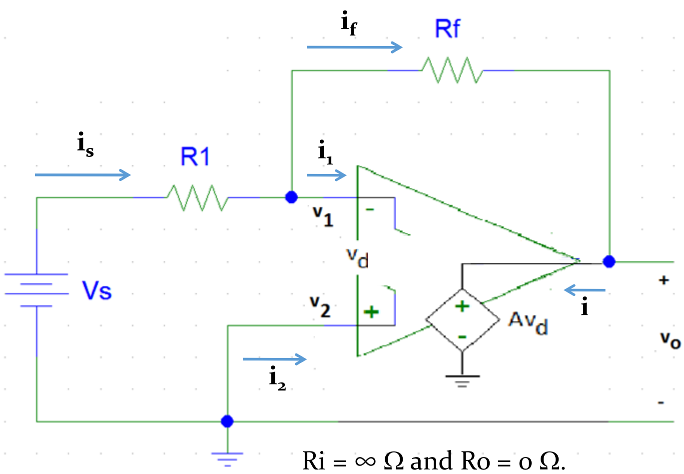
- The non-inverting Amplifier.
- The inverting Amplifier

(Note: Negative feedback is used to limit the gain)

Example #2: Closed Loop Gain Av (inverting amplifier)

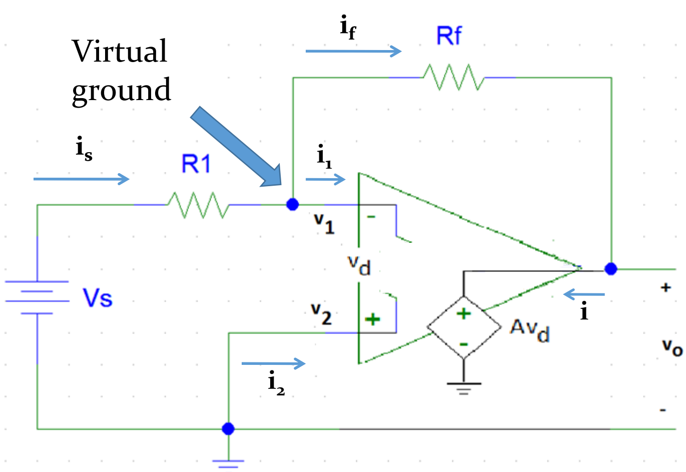


Example #2 (con't)

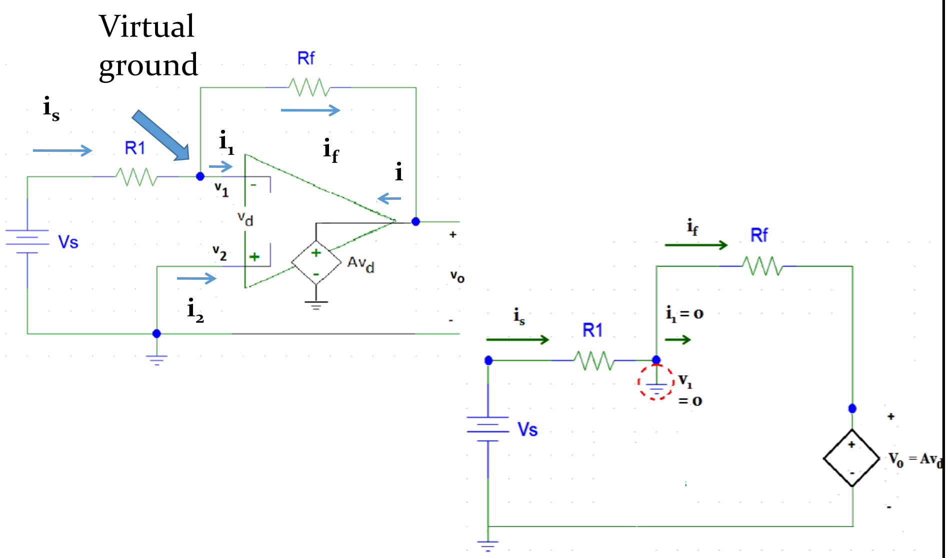


Ri = ∞ Ω and Ro = 0 Ω.

The op amp outputs a voltage Vo such that V1 = V2.
V2=Vp=0 ➔ V1=Vn=0
(virtual ground – potential equal 0 even though it is not connected really to ground)



Example #2 (con't)



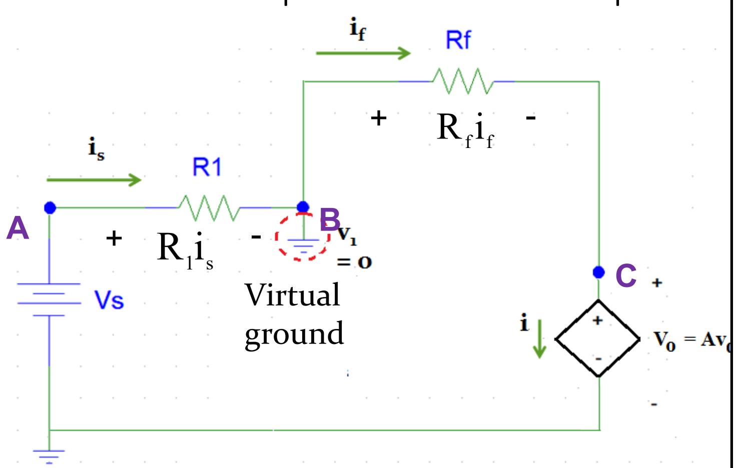
Example #2: Closed Loop Gain

$$V_2 = 0V \Rightarrow V_1 = 0V$$

$$V_S = R_1 i_s$$

$$v_o = -R_f i_f$$

$$i_s = i_f = i$$

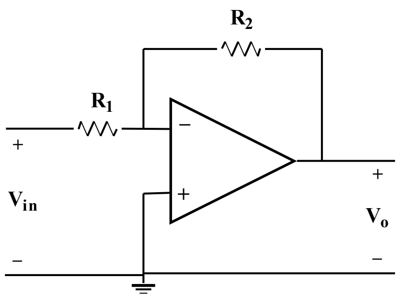


$$A_V = V_o / V_s = \frac{-R_f i_f}{R_1 i_s}$$

$A_V = -R_f / R_1$ ➡ This circuit is known as an inverting amplifier.

Inverting Amplifier (previous example of slide 30)

Find V_o in terms of V_{in} for the following configuration.



$$V_0 = \frac{-R_2}{R_1} V_{in}$$

With $R_2 = 40\ k\Omega$ and $R_1 = 10\ k\Omega$, we have

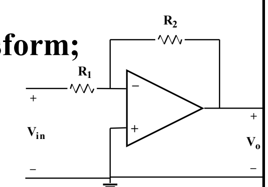
$$V_o = -4V_{in}$$

Earlier we got →

$$\boldsymbol{V_o = -3.99 V_{in}}$$

Inverting Op Amp:

When $V_i = 0$ in and we apply the Laplace Transform;



$$\frac{V_0(s)}{V_{in}(s)} = \frac{-R_2}{R_1}$$

In fact, we can replace R_2 with $Z_{fb}(s)$ and R_1 with $Z_1(s)$ and we have the important expression;

$$\frac{V_0(s)}{V_{in}(s)} = \frac{-Z_{fb}(s)}{Z_{in}(s)}$$

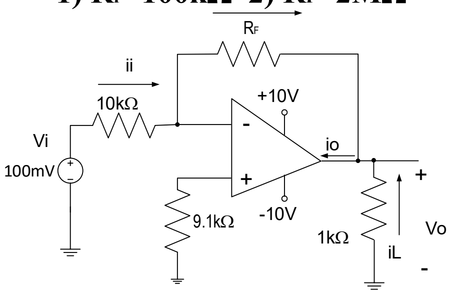
At this point in circuits we are not able to appreciate the use of this equation. We will revisit this at a later point in circuits but for now we point out that judicious selections of $Z_{fb}(s)$ and $Z_{in}(s)$ leads to important applications in

- **Analog Compensators in Control Systems**
- **Analog Filters**
- **Application in Communications**

Example

Find the value of Vo and Io and verify if the opamp is in linear or saturation mode for two values of feedback resistor; assume Io(max)=20 mA:

1) R_F=100kΩ 2) R_F=2MΩ



$$V_p = V(+) = 0V$$

$$i_1 = \frac{V_1}{R_1}$$

$$i_i = i_f = \frac{V_i}{10k} = \frac{100\,\text{mV}}{10\,\text{k}\Omega} = 10\,\mu\text{A}$$

$$1)\ V_o = -\frac{R_F}{10\,\text{k}\Omega} V_i = -10\,V_i = -1\text{V}$$

$$V_o > -V_{sat}$$

Linear mode

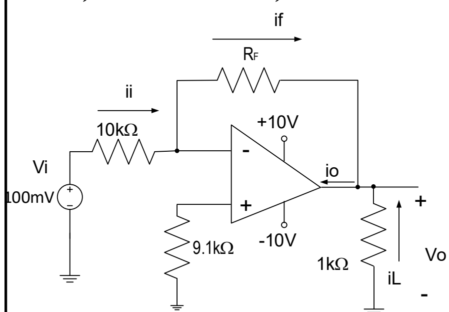
<u>Important</u>

Io(max) is few mA for most opamps which limits the values of resistors to be used to kohm range

Example

Find the value of Vo and Io and verify if the opamp is in linear or saturation mode for two values of feedback resistor:

1) R_F=100kΩ 2) R_F=2MΩ



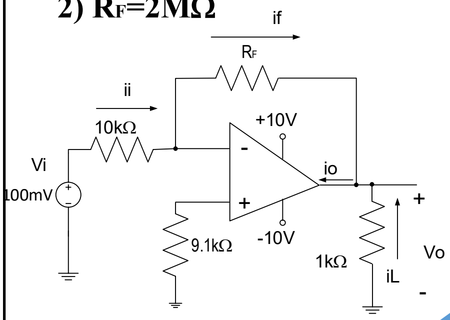
$$i_o = i_f + i_L$$

$$i_L = \frac{1V}{1k} = 1\text{mA}$$

$$i_o = 10\mu\text{A} + 1\text{mA} = 1.01\text{mA} < \text{Io(max)}$$

Example continued

2) R_F=2MΩ



$$i_i = i_f = \frac{V_i}{10k} = \frac{100mV}{10k} = 10\ \mu A$$

$$2) V_o = -\frac{R_F}{10k} V_i = -200 V_i = -20V$$

$$V_o < -V_{sat}$$

Saturation mode $\Rightarrow$ V_o is limited to - V_{sat}

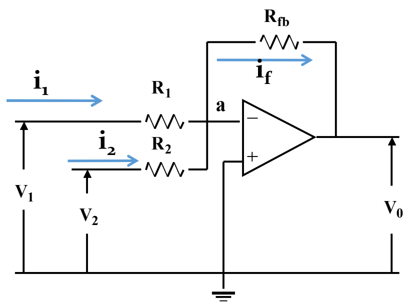
$$\therefore V_o = -V_{sat} = -8V$$

$$i_L = \frac{8V}{1k} = 8mA$$

$$i_o = 10\ \mu A + 8mA = 8.01mA < Io(max)$$

Inverting Adder or Summing Amplifier

Summing Amplifier: This is an application of inverting amplifier



$$V_p = V_{(+)} = 0V$$

$$i_1 = \frac{V_1}{R_1} \qquad i_2 = \frac{V_2}{R_2}$$

$$V_o = -R_f i_f$$

$$i_f = i_1 + i_2$$

$$V_o = -R_f [i_1 + i_2]$$

$$V_0 = -\left[\left(\frac{R_{fb}}{R_1}\right)V_1 + \left(\frac{R_{fb}}{R_2}\right)V_2\right]$$

If $R_1 = R_2 = R_{fb}$ then,

$$V_0 = -[V_1 + V_2]$$

Therefore, we can add signals with an op amp

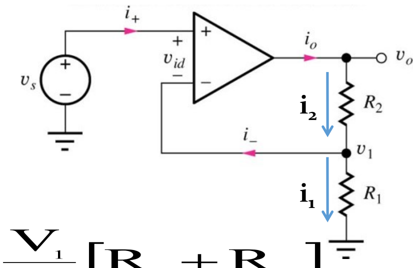
The non-inverting op amp.

The non-inverting op amp has the input voltage connected to its (+) terminal while no voltage at the negative terminal

$$V_n = V_1 = V_p = V_s$$

$$i_+ = i_- = 0$$

$$V_o = V_{R1} + V_{R2}$$



$$V_{R1} = V_1 = i_1 R_1$$

$$V_{R2} = i_2 R_2$$

but

$$i_1 = i_2 = \frac{V_1}{R_1}$$

$$V_o = \frac{V_1}{R_1}[R_1 + R_2]$$

which gives,

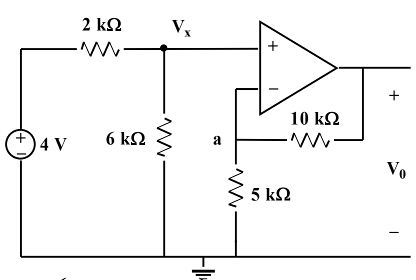
$$\boxed{V_o = \left(1 + \frac{R_2}{R_1}\right) V_s}$$

Example: Non-inverting Amplifiers

Example: Find V_0 for the following op amp configuration.

$$V_x = \frac{6k}{6k + 2k} 4V$$

$$V_x = 3V$$



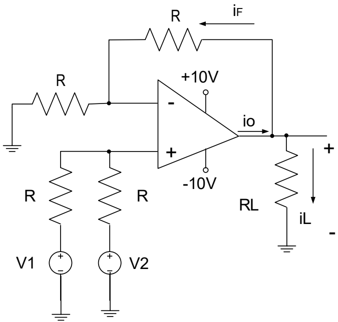
$$V_o = \left(1 + \frac{R_F}{R_i}\right) V_x$$

$$V_o = \left(1 + \frac{10k}{5k}\right) 3V$$

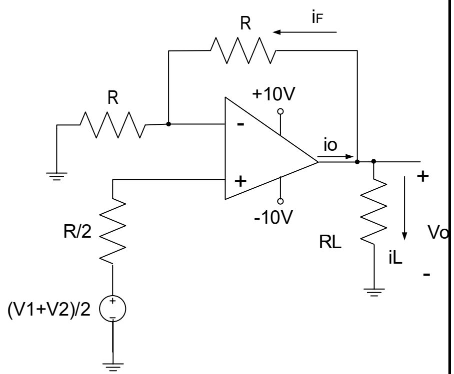
$$V_o = 9V$$

Make sure that: $-V_{sat} < V_o < +V_{sat}$

Non-Inverting Adder



==>
Thevenin's equivalent at + terminal



$$V_{TH} = \left(\frac{R}{R+R}\right)V_1 + \left(\frac{R}{R+R}\right)V_2$$
$$= \left[\frac{V_1+V_2}{2}\right]$$

$$V_+ = V_{TH} \quad \Leftarrow \quad (\text{since } i_+ = 0)$$

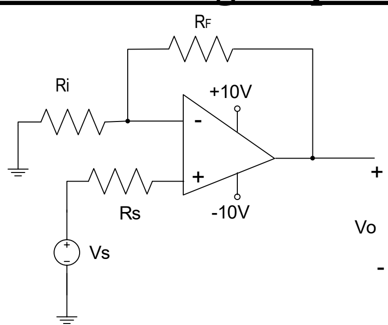
$$V_o = \left(1+\frac{R}{R}\right)V_+$$

$$V_o = \left(1+\frac{R}{R}\right)\left[\frac{V_1+V_2}{2}\right]$$

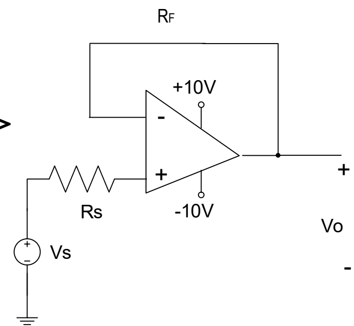
$$V_o = 2\left[\frac{V_1+V_2}{2}\right]$$

$$V_o = \left[V_1+V_2\right]$$

Non-Inverting Amplifier with unity Gain (Buffer)



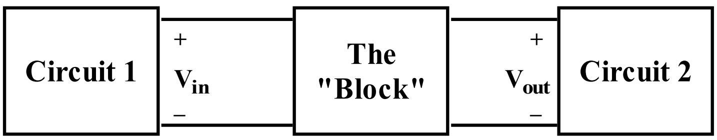
==>



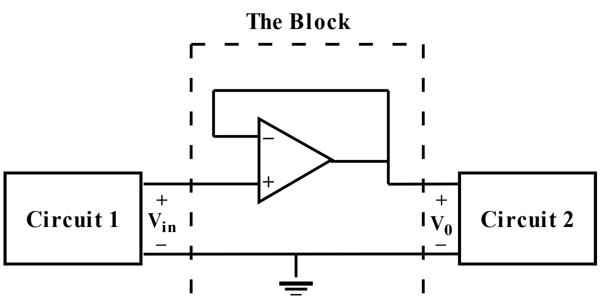
$$V_o = V(-)$$
$$V(-) = V(+) = Vs$$
$$V_o = Vs$$
$$\frac{V_o}{Vs} = 1$$

Buffer or Isolation Amplifier or Voltage Follower

- Applications arise in which we wish to connect one circuit to another without the first circuit loading the second.
- This requires that we connect to a "block" that has infinite input impedance and zero output impedance.
- An operational amplifier does a good job of approximating this.
- Consider the following:



Buffer or Isolation Amplifier or Voltage Follower

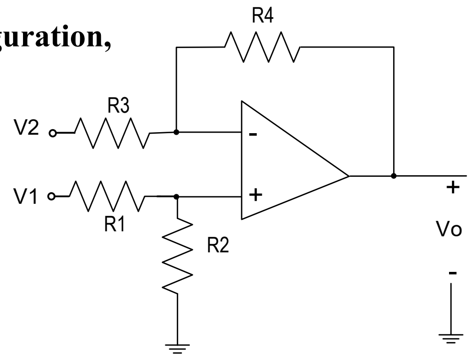


Circuit isolation with an op amp.

It is easy to see that: $V_0 = V_{in}$

Difference Amplifier (Subtractor)

For the following op amp configuration, in order to find Vo we can use Superposition:



1) Short V2 and find contribution of V1 to Vo ==> non-inverting amp

$$V_o = V_{o1}$$

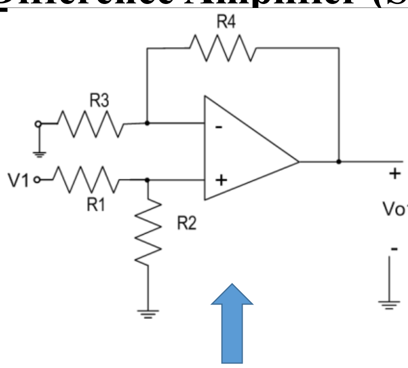
2) Short V1 and find contribution of V2 to Vo ==> inverting amp

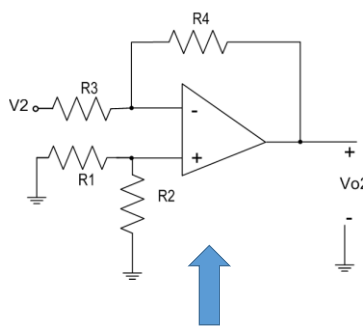
$$V_o = V_{o2}$$

3) The total output is found by summing the two results above

$$V_o = V_{o1} + V_{o2}$$

Difference Amplifier (Subtractor)





Non-Inverting Amplifier

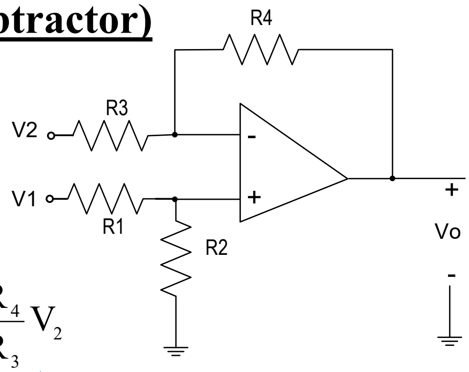
$$V_{o1} = \left(1 + \frac{R_4}{R_3}\right) V_+$$

$$V_{o1} = \left(1 + \frac{R_4}{R_3}\right)\left(\frac{R_2}{R_1 + R_2}\right) V_1$$

Inverting Amplifier

$$V_{o2} = -\frac{R_4}{R_3} V_2$$

Difference Amplifier (Subtractor)



$$V_o = \left(1+\frac{R_4}{R_3}\right)\left(\frac{R_2}{R_1+R_2}\right)V_1 - \frac{R_4}{R_3}V_2$$

$$V_o = aV_1 - bV_2$$

$$a = \left(1+\frac{R_4}{R_3}\right)\left(\frac{R_2}{R_1+R_2}\right)$$

$$b = \frac{R_4}{R_3}$$

let $R_1 = R_3 = R$

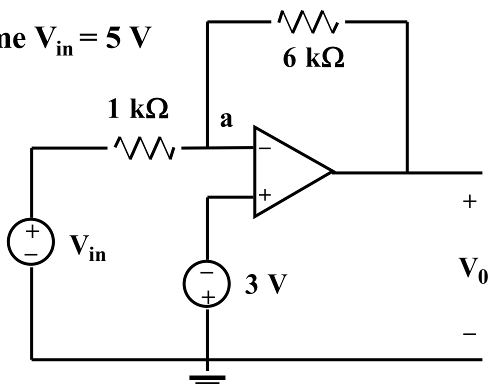
$R_2 = R_4 = mR$

$\therefore\ V_o = m(V_1 - V_2)$

under this condition this is a difference amplifier

Example **Consider the op amp configuration below.**

Assume V_{in} = 5 V

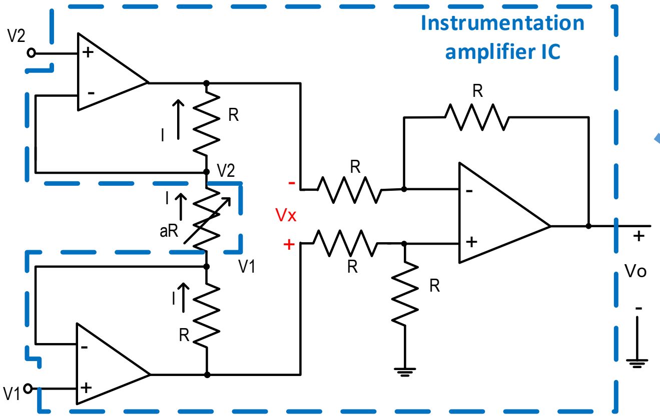


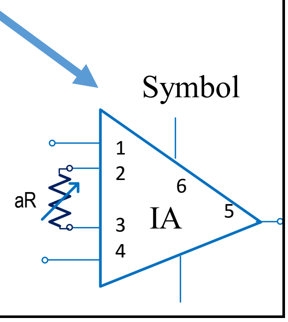
$$V_0 = (1+\frac{6k}{1k})(-3) - (\frac{6k}{1k})(5)$$
$$= -21 - 30 = -51V$$

Since V_0 = -51 V **(op amp will saturate and Vo will be limited to -Vsat)**

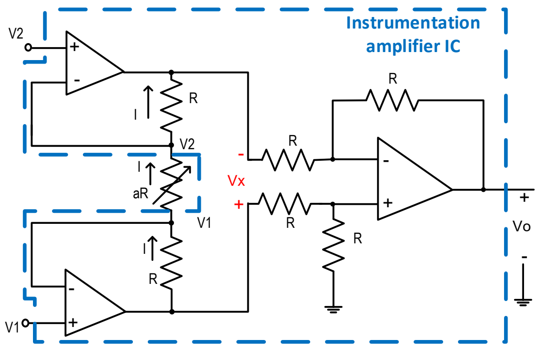
Instrumentation Amplifier

- The previous difference amplifier has low input impedance and it is difficult to vary the gain "m"
- The instrumentation amplifier solves this problem by adding a buffer stage and a difference amplifier stage to solve the disadvantages of difference amplifier





Instrumentation Amplifier



assume $V_1 > V_2$

$$I = \frac{V_1 - V_2}{aR}$$

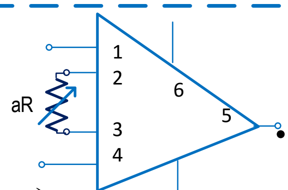
$$V_x = (R + R + aR)I$$

$$V_x = (R + R + aR)\frac{V_1 - V_2}{aR}$$

$$V_x = \left(\frac{(2+a)R}{aR}\right)(V_1 - V_2)$$

$$V_x = \left(1 + \frac{2}{a}\right)(V_1 - V_2)$$

$$V_O = mV_x$$



for $m = 1$

$$V_o = V_x = \left(1 + \frac{2}{a}\right)(V_1 - V_2)$$

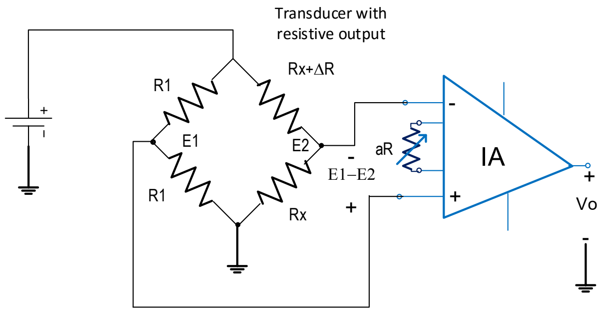
- Only by varying the value of potentiometer aR , the output can be adjusted
- R is an internal resistance given in data sheet of " IA" IC

Example: Instrumentation Amplifier

$$V_x = \left(1 + \frac{2}{a}\right)\left(E_1 - E_2\right)$$

$$E_1 = \frac{R_1}{R_1 + R_1} E = \frac{1}{2} E$$

$$E_2 = \frac{R_x}{R_x + R_x + \Delta R_x} E$$



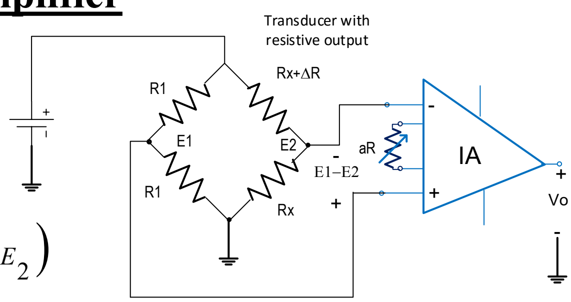
$$E_1 - E_2 = \frac{1}{2} E - \frac{R_x}{2R_x + \Delta R_x} E$$

$$= \frac{2R_x + \Delta R_x}{2\left(2R_x + \Delta R_x\right)} E - \frac{2R_x}{2\left(2R_x + \Delta R_x\right)} E$$

$$E_1 - E_2 = \frac{E}{2}\left(\frac{2R_x + \Delta R_x - 2R_x}{2R_x + \Delta R_x}\right)$$

$$= \frac{E}{2}\left(\frac{\Delta R_x}{2R_x + \Delta R_x}\right)$$

Instrumentation Amplifier



assume $\Delta R_x << R_x$

$$V_o = E1 - E2 = \left(1 + \frac{2}{a}\right)\left(E_1 - E_2\right)$$

$$= \left(1 + \frac{2}{a}\right)\frac{E}{4R_x}\left(\Delta R_x\right)$$

$V_o \;\alpha\; \Delta R_x$

let $R_1 = 10k$; $E = 10V$; $\Delta R_x = 10$;

$aR = 50.12$ ohm

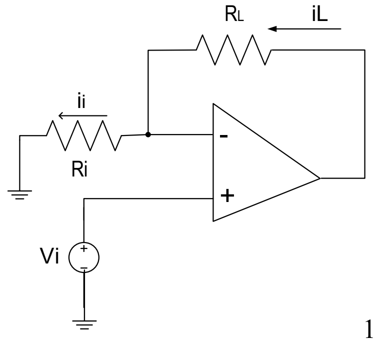
(potentiomater is set to 50.12 ohms)

$$a = \left(\frac{aR}{R}\right) = \left(\frac{50.12}{10000}\right) = 0.005013$$

$$\left(1 + \frac{2}{a}\right) = 400$$

$$V_o = (400)\left(\frac{10}{4}\right) * \frac{10}{10000} = 1\,V$$

Voltage to Current converter



$$i_i = \frac{V_i}{R_i}$$

$$i_i = i_L$$

let $V_i = 1V; Ri = 1k$

$$i_L = \frac{1V}{1k} = +1mA$$

$$i_L = \frac{1V}{1k} = +1mA$$

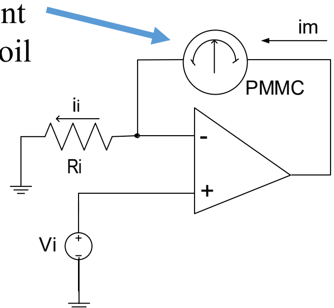
let $V_i = -1V; Ri = 1k$

$$i_L = \frac{-1V}{1k} = -1mA$$

Here we converted ± 1V to ± 1mA

Voltage to Current converter

PMMC: Permanent magnet moving coil



$$i_i = \frac{V_i}{R_i}$$

$$i_i = i_m$$

let $V_i = \pm 1V; Ri = 1k$

$$i_m = \frac{1V}{1k} = \pm 1mA$$

Current to Voltage converter

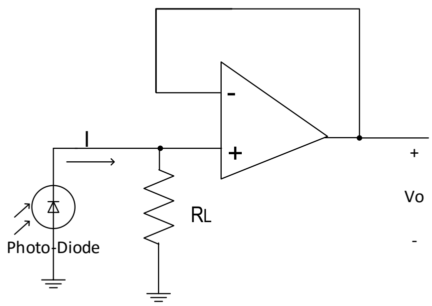


Photo-diode
Is a diode which is biased by certain type of light

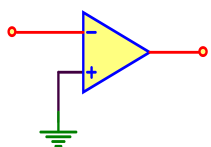
$V_{(+)} = I.R_L$

$V_o = V_{(-)} = V_{(+)} = I.R_L$

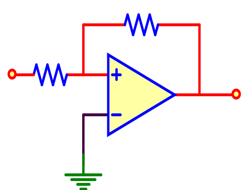
Here we converted current I to voltage Vo

I - can be any current source, sensor or device with current output

Non – Linear Applications of opamps



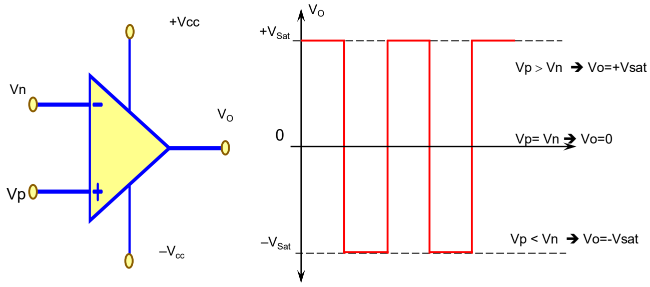
(a) No Feedback (open loop comparator circuit)



(b) Positive Feedback

- No feedback : Open loop (used in comparators)- non linear application
- Positive feedback : Feedback to the non inverting input (Used in oscillators) and Schmitt triggers (comparators with hysterisis)-non Linear Application

OP AMP as a Comparator (compares 2 voltages and produces a signal to indicate which is greater)

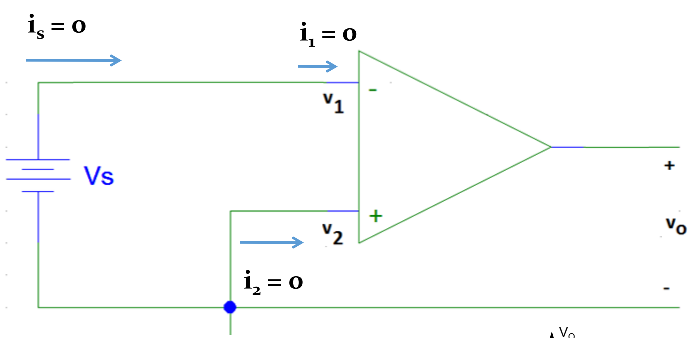


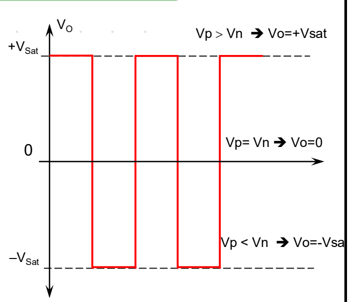
(a) Comparator Circuit **(b)** Comparator Output

Applications of Comparators

- Analog to digital converters (ADC)
- Counters (e.g. count pulses that exceed a certain voltage level).
- Cross Over Detectors

Example #1: Voltage Comparator





(b) Comparator Output

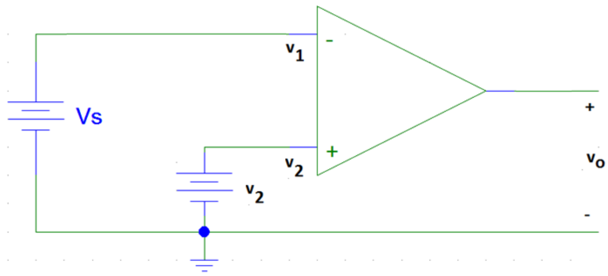
When Vs is equal to 0V, Vo = 0V.

When Vs is smaller than 0V, (V2>V1 ==> Vo = +Vsat)

When Vs is larger than 0V, ((V2<V1 ==> Vo = -Vsat)

Example

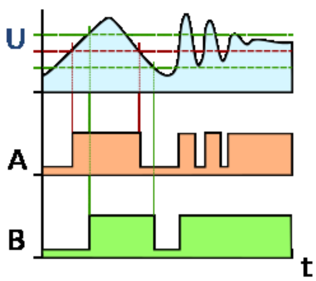
- Given how an op amp functions, what do you expect Vo to be if v2 = 5V when:
  1. Vs = 0V?
     Answer Vo=+Vsat
  2. Vs = 5V?
     Answer Vo=0 (practically impossible to have both V1=V2)
  3. Vs = 6V?
     Answer Vo=-Vsat



a **Schmitt trigger** is a comparator circuit with hysteresis,

Schmitt trigger devices are typically used in signal conditioning applications to remove noise from signals used in digital circuits, particularly mechanical switch bounce.
They are also used in closed loop negative feedback configurations to implement relaxation oscillators, used in function generators and switching power supplies.

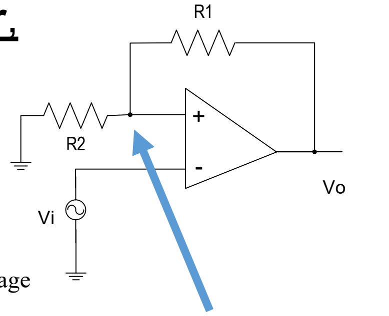
The output of a Schmitt trigger (B) and a comparator (A), when a noisy signal (U) is applied.
The green dotted lines are the circuit's switching thresholds.
The Schmitt trigger tends to remove noise from the signal.



Schmitt trigger,

This is a comparator circuit and the output is $Vo = \pm \text{Vsat}$

Analysis: step 1



R1 is Fed back from output to (+) input
This is called positive feedback

$\text{let } V_o = +\text{Vsat}$

$V_+ = \frac{R_2}{R_1 + R_2} + \text{Vsat} = V_{UT}$ - upper threshold voltage

in order for Vo to be + Vsat

$Vd > 0$

$Vd = V(+) - V(-) > 0$

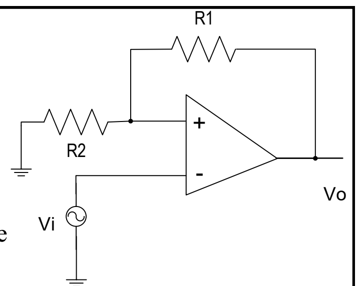
$\frac{R_2}{R_1 + R_2} + \text{Vsat} > V_i$

when $V_{UT} > V_i \Rightarrow Vo = +\text{Vsat}$

But when $V_i > V_{UT} \Rightarrow$ Vo switches to $-$ Vsat

Schmitt trigger,

Analysis: step 2



$\text{let } V_o = -\text{Vsat}$

$V_+ = \frac{R_2}{R_1 + R_2} - \text{Vsat} = V_{LT}$ - Lower threshold voltage

in order for Vo to be $-$ Vsat

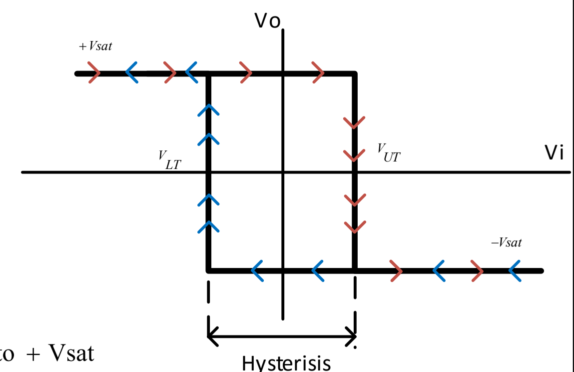
$Vd < 0$

$Vd = V(+) - V(-) < 0$

$\frac{R_2}{R_1 + R_2} - \text{Vsat} < V_i$

when $V_{LT} < V_i \Rightarrow Vo = -\text{Vsat}$

But when $V_i < V_{LT} \Rightarrow$ Vo switches to + Vsat



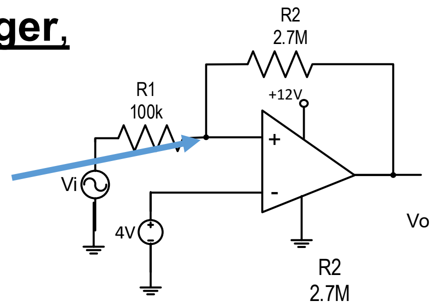
$\left| V_{UT} - V_{LT} \right| = \text{Hysterisis}$

Schmitt trigger,

Example: Find and sketch Vo(t) and the plot of Vo=f(Vi)

Solution: this is a Schmitt trigger and

$$Vo = \pm \text{Vsat}$$



1) let $V_o = +Vsat$

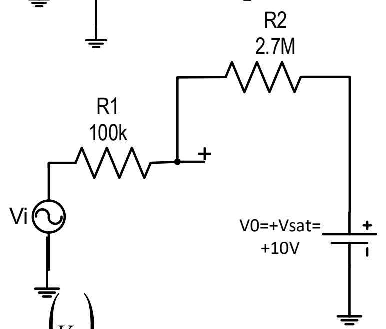
in order for Vo to be + Vsat

$\text{Vd} > 0$

$\text{Vd} = \text{V}(+) - \text{V}(-) > 0$

$\text{V}(+) > \text{V}(-)$

$\text{V}(-) = 4V$



$$V(+) = \frac{100k}{(2700+100)k}\left(+Vsat\right) + \frac{2700k}{(2700+100)k}\left(V_i\right)$$

$$\frac{100k}{(2700+100)k}\left(+Vsat\right) + \frac{2700k}{(2700+100)k}\left(V_i\right) > 4V$$

$$\frac{100\,k}{(2700\,+100\,)k}\left(10V\right) + \frac{2700\,k}{(2700\,+100\,)k}\left(V_i\right) > 4V$$

$$\left(V_i\right) > \left[4V - \left(\frac{100\,k}{(2700\,+100\,)k}\left(10V\right)\right)\right]\left[\frac{(2700\,+100\,)k}{2700\,k}\right] \Longrightarrow V_i > 3.777\ V$$

when $V_i > 3.777 \Rightarrow \text{Vo} = +Vsat$; But when $V_i < 3.777 \Rightarrow$ Vo switches to $-Vsat$

2) let $V_o = -Vsat = (0+2) = 2V$

in order for Vo to be - Vsat ==> $\text{Vd} < 0$; $\therefore \text{V}(+) < \text{V}(-)$

$$V(+) = \frac{100k}{(2700+100)k}\left(-Vsat\right) + \frac{2700k}{(2700+100)k}\left(V_i\right)$$

$$\frac{100k}{(2700+100)k}\left(-Vsat\right) + \frac{2700k}{(2700+100)k}\left(V_i\right) < 4V$$

$$\left(V_i\right) < \left[4V - \left(\frac{100k}{(2700+100)k}\left(2V\right)\right)\right]\left[\frac{(2700+100)k}{2700k}\right] \Longrightarrow V_i < 4.074\,V$$

when $V_i < 4.074\,V \Rightarrow \text{Vo} = -Vsat$; But when $V_i > 4.074 \Rightarrow$ Vo switches to $+Vsat$

Conclusion

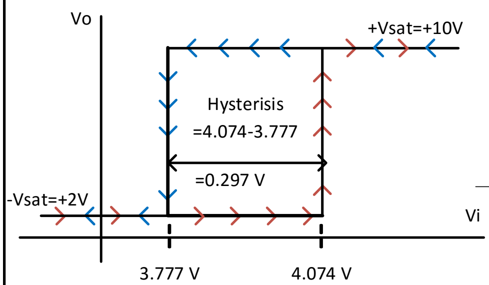
1) when Vi is decreasing, as long as Vi is $> 3.777\text{V} \Rightarrow \text{Vo} = +Vsat$;

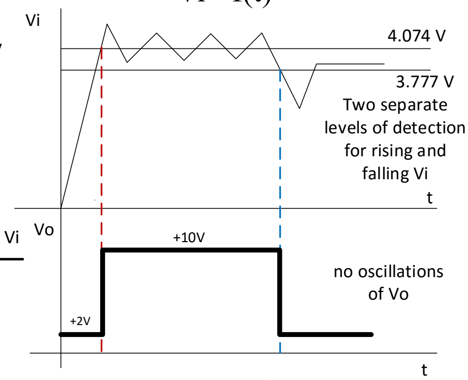
but when V_i becomes $< 3.777 \Rightarrow$ Vo switches to $-Vsat$

2) when Vi is increasing, as long as Vi is $< 4.074\text{V} \Rightarrow \text{Vo} = -Vsat$;

but when V_i becomes $> 4.074 \Rightarrow$ Vo switches to $+Vsat$

Vi =f(t)

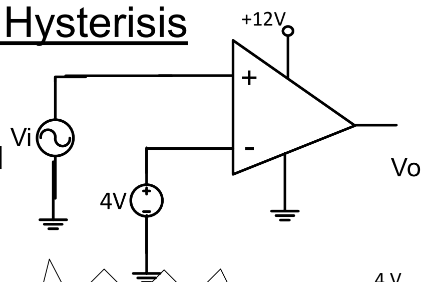




Vo =f(Vi)

Vo =f(t)

Comparator without Hysterisis



Example: Find and sketch Vo(t) and the plot of Vo=f(Vi)

Solution: this is a comparator and

$$Vo = \pm \text{Vsat}$$

1) in order for Vo to be + Vsat

$\text{Vd} > 0$ and $\text{Vd} = \text{V}(+) - \text{V}(-) > 0$

$\text{V}(+) > \text{V}(-)$

$\text{V}(-) = 4V; V(+) = V_i$

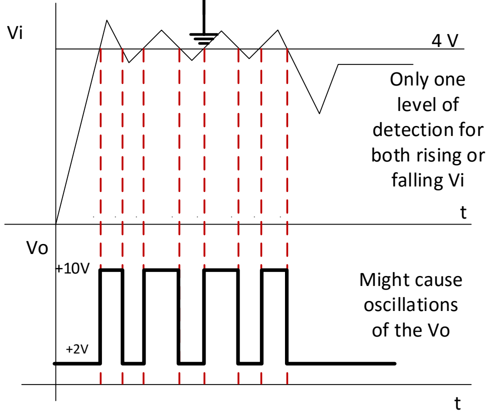
$V_i > 4V$

2) in order for Vo to be $-$ Vsat = 2V

$\text{Vd} < 0$ and $\text{Vd} = \text{V}(+) - \text{V}(-) < 0$

$\text{V}(+) < \text{V}(-)$

$V_i < 4V$



Integrator

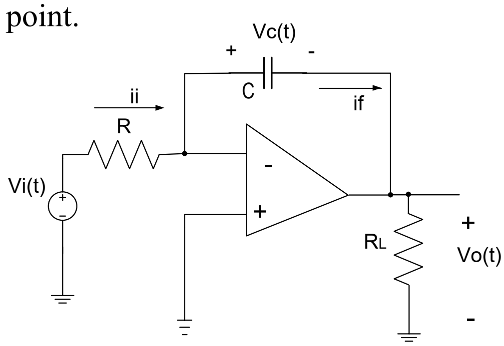
- So far, the input and feedback components have been resistors. If the feedback component used is a capacitor,, the resulting connection is called an *integrator.*
- Recall that virtual ground means that we can consider the voltage at the junction of R and X_c to be ground (since $V_+ = 0$ V) but that no current goes into ground at that point.

$$i_i = i_f = \frac{V_i}{R}$$

$$Vc(t) = \frac{1}{C}\int_0^t i_f(t)dt$$

$$V_o = -Vc(t)$$

$$Vo = -\frac{1}{C}\int_0^t \frac{V_i(t)}{R_i}dt = -\frac{1}{RC}\int_0^t V_i(t)dt$$



Differentiator

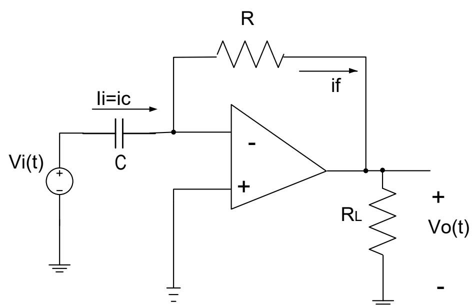
A differentiator ,while not as useful as the circuit forms covered above, the differentiator does provide a useful operation, the resulting relation for the circuit being

$$i_i = i_f = i_C = C\frac{dV_i(t)}{dt}$$

$$Vc(t) = \frac{1}{C}\int_0^t i_f(t)dt$$

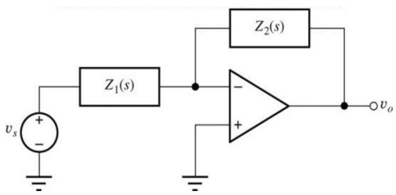
$$V_o = -i_f(t)R$$

$$Vo = -\left(C\frac{dV_i(t)}{dt}\right)(R) = -RC\frac{dV_i(t)}{dt}$$



The Active Low-pass Filter

The gain analysis of this inverting amplifier. Note s = jω.



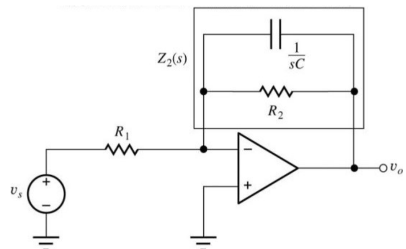
$$A_v = \frac{\tilde{v}_o(j\omega)}{\tilde{v}_s(j\omega)} = -\frac{Z_2(j\omega)}{Z_1(j\omega)} \qquad Z_1(j\omega) = R_1$$

$$Z_2(j\omega) = \frac{R_2 \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} = \frac{R_2}{j\omega C R_2 + 1}$$

$$A_v = -\frac{R_2}{R_1} \frac{1}{(1 + j\omega C R_2)}$$

$$= \frac{-K}{\left(1 + \frac{j\omega}{\omega_c}\right)}$$

$$K = \frac{R_2}{R_1}$$



& cut-off frequency: $\omega_c = 2\pi f_c = \frac{1}{R_2 C} \quad \therefore f_c = \frac{1}{2\pi R_2 C}$

Frequency Response (Bode Plot)

Magnitude Plot (Magnitude in decibels vs log of frequency)

$$A_{dB} = 20\log|H(j\omega)|$$

$$|H(j\omega)| = |A_v(j\omega)|$$

$$|A_v(j\omega)| = \frac{K}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$$

at $\omega = 0.1\omega_c$

$$20\log \frac{1}{\sqrt{1 + \left(\frac{0.1\omega_c}{\omega_c}\right)^2}} = 20\log \frac{1}{\sqrt{1 + 0.01}}$$

$$\cong 20\log 1 = 0dB$$

at $\omega = \omega_c$

$$20\log \frac{1}{\left|\sqrt{1 + \left(\frac{\omega_c}{\omega_c}\right)^2}\right|} = 20\log \frac{1}{\left|\sqrt{1+1}\right|}$$

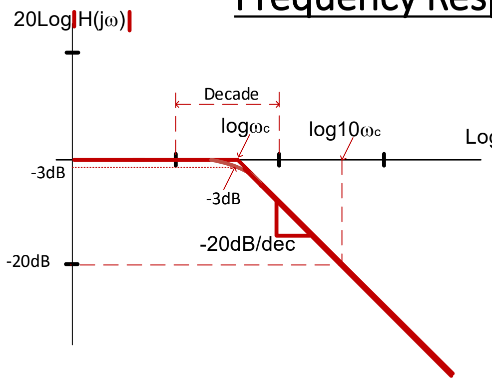
$$= 20\log|0.707| = -3dB$$

at $\omega = 10\omega_c$

$$20\log \frac{1}{\left|\sqrt{1 + \left(\frac{10\omega_c}{\omega_c}\right)^2}\right|} = 20\log \frac{1}{\left|\sqrt{1+100}\right|}$$

$$= 20\log|0.1| = -20dB$$

Frequency Response (Bode Plot)



Summary

at $\omega = 0.1\omega_c$

$= 20\log 1 = 0dB$

at $\omega = \omega_c$

$= 20\log 0.707 = -3dB$

at $\omega = 10\omega_c$

$= 20\log 0.1 = -20dB$

at $\omega = 100\omega_c$

$= 20\log 0.01 = -40dB$

- At frequencies below ω_c, the amplifier is an inverting amplifier with gain set by the ratio of resistors R_2 and R_1.
- At frequencies above ω_c, the amplifier response "rolls off" at -20dB/decade.
- Notice that cutoff frequency and gain can be independently set.

Active Low-pass Filter: Example

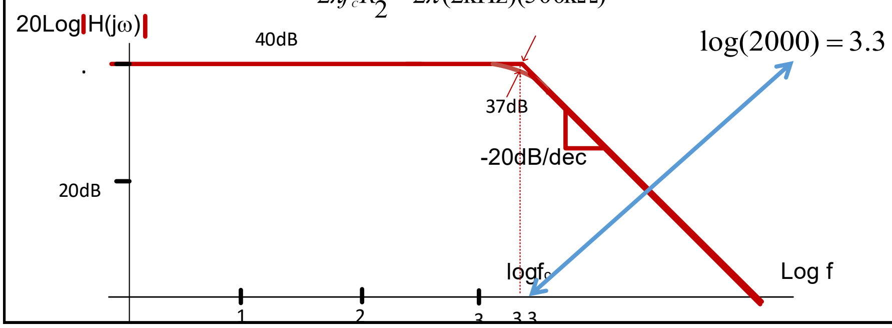
- **Problem**: Design an active low-pass filter
- **Given Data**: A_v= 40 dB, R_{in}= 5 kΩ, f_c = 2 kHz
- **Assumptions:** Ideal op amp, specified gain represents the desired low-frequency gain.
- **Analysis**: $|A_v| = 10^{40\text{dB}/20\text{dB}} = 100$

  Input resistance is controlled by R_1 and voltage gain is set by R_2 / R_1.

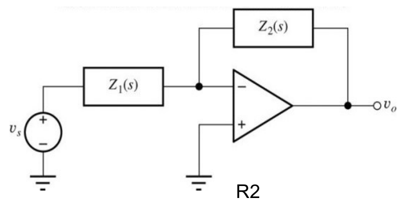
  The cutoff frequency is then set by C.

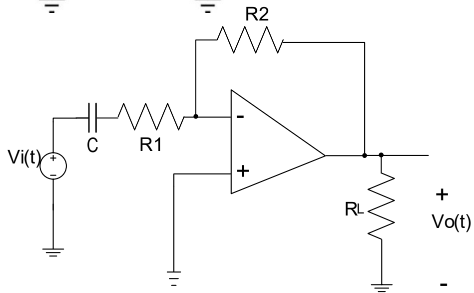
$$R_1 = R_{in} = 5\text{k}\Omega \quad \text{and} \quad |A_v| = \frac{R_2}{R_1} \Rightarrow R_2 = 100R_1 = 500\text{k}\Omega$$

$$C = \frac{1}{2\pi f_c R_2} = \frac{1}{2\pi(2\text{kHz})(500\text{k}\Omega)} = 159\text{pF}$$



The Active High-pass Filter





The gain analysis of this inverting amplifier.

$$A_v = \frac{\tilde{v}_o(j\omega)}{\tilde{v}_s(j\omega)} = -\frac{Z_2(j\omega)}{Z_1(j\omega)} \qquad Z_2(j\omega) = R_2$$

$$Z_1(j\omega) = R_1 + \frac{1}{j\omega C} \qquad A_v = -\frac{R_2}{(R_1 + \frac{1}{j\omega C})}$$

$$A_v = -\frac{R_2/R_1}{(1 + \frac{1}{j\omega R_1 C})}$$

$$= \frac{-K}{\left(1 + \frac{\omega_c}{j\omega}\right)}$$

$$K = \frac{R_2}{R_1} \text{ high frequency gain}$$

$$\omega_c = 2\pi f_c = \frac{1}{R_1 C} \quad \therefore f_c = \frac{1}{2\pi R_1 C}$$

Frequency Response (Bode Plot)

Magnitude Plot (Magnitude in decibels vs log of frequency)

$$A_{dB} = 20\log|H(j\omega)|$$

$$|H(j\omega)| = |A_V(j\omega)|$$

$$= \frac{K}{\sqrt{1 + \left(\frac{\omega_c}{\omega}\right)^2}}$$

at $\omega = 0.1\omega_c$

$$20\log\frac{1}{\sqrt{1 + \left(\frac{\omega_c}{0.1\omega_c}\right)^2}} = 20\log\frac{1}{\sqrt{1+100}}$$

$$\cong 20\log 0.01 = -20dB$$

at $\omega = \omega_c$

$$20\log\frac{1}{\left|\sqrt{1 + \left(\frac{\omega_c}{\omega_c}\right)^2}\right|} = 20\log\frac{1}{|\sqrt{1+1}|}$$

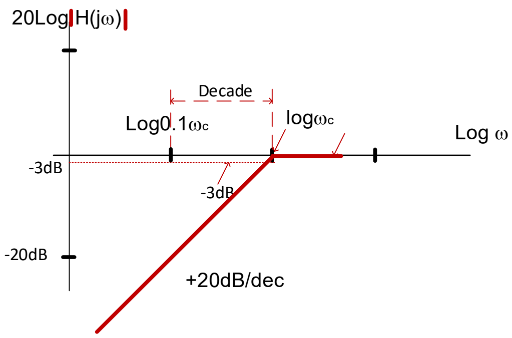
$$= 20\log|0.707| = -3dB$$

at $\omega = 0.01\omega_c$

$$20\log\frac{1}{\left|\sqrt{1 + \left(\frac{\omega_c}{0.01\omega_c}\right)^2}\right|} = 20\log\frac{1}{|\sqrt{1+10000}|}$$

$$= 20\log 0.01 = -40dB$$

Frequency Response (Bode Plot)



Summary

at $\omega = 0.1\omega_c$

$= 20\log 0.1 = -20dB$

at $\omega = \omega_c$

$= 20\log 0.707 = -3dB$

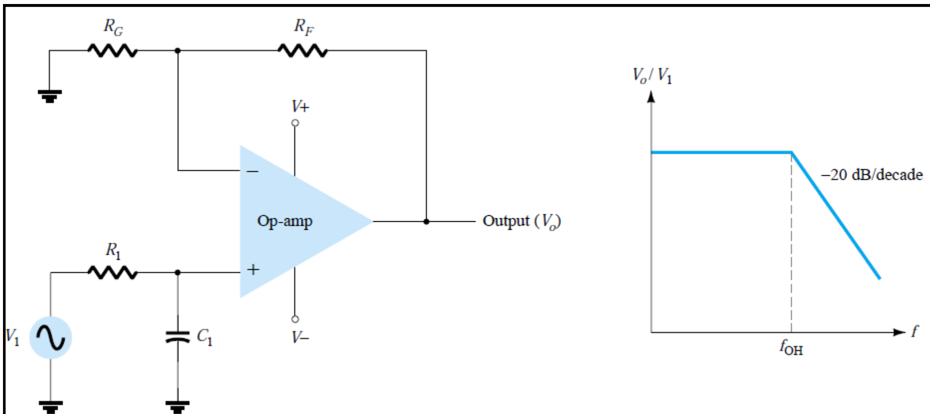
at $\omega = 0.01\omega_c$

$= 20\log 0.01 = -40dB$

at $\omega = 0$

$= 20\log 1 = 0dB$

- At frequencies above ω_c, the amplifier is an inverting amplifier with gain set by the ratio of resistors R_2 and R_1.
- At frequencies below ω_c, the amplifier response "rolls off" at -20dB/decade.
- Notice that cutoff frequency and gain can be independently set.

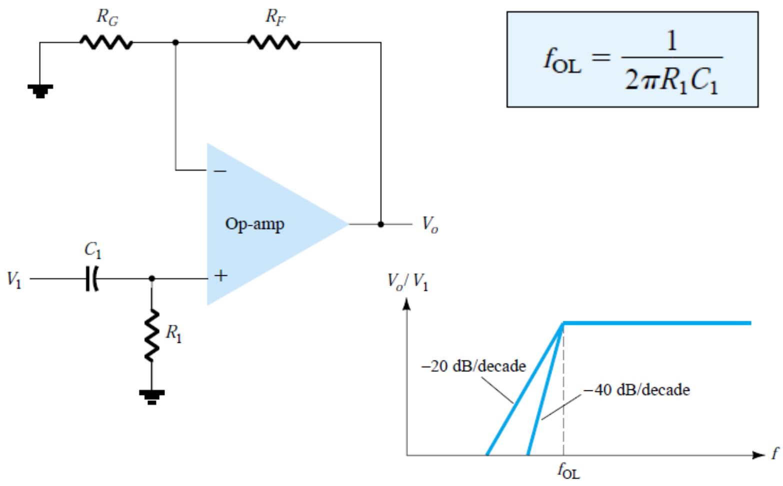


$$A_V = 1 + \frac{R_F}{R_G}$$

at a cutoff frequency of

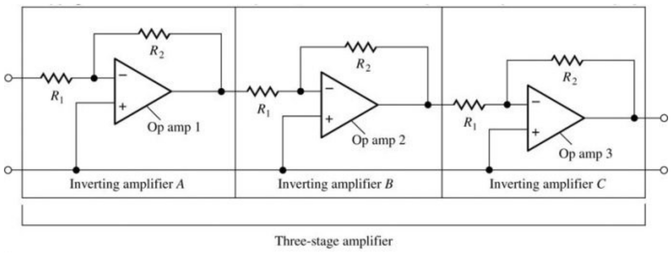
$$f_{OH} = \frac{1}{2\pi R_1 C_1}$$

High-Pass Active Filter

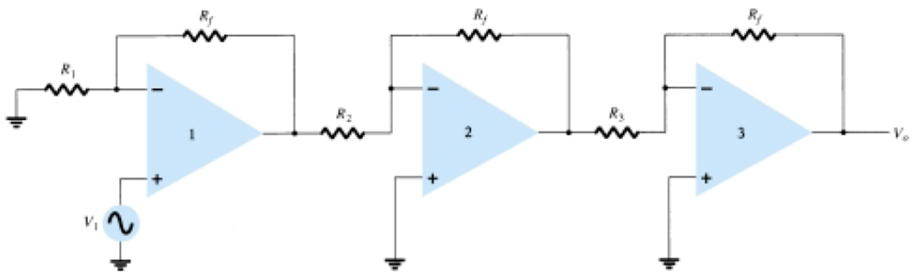


$$f_{OL} = \frac{1}{2\pi R_1 C_1}$$

Cascaded Amplifiers



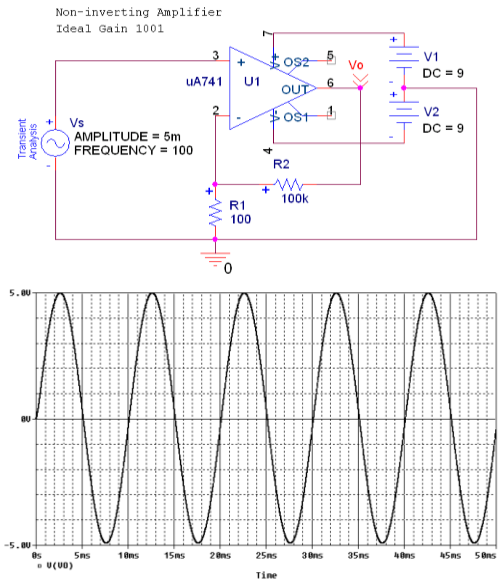
- Connecting several amplifiers in cascade (output of one stage connected to the input of the next) can meet design specifications not met by a single amplifier.
- Each amplifer stage is built using an op amp with parameters A, R_{id}, R_o, called open loop parameters, that describe the op amp with no external elements.
- A_v, R_{in}, R_{out} are closed loop parameters that can be used to describe each closed-loop op amp stage with its feedback network, as well as the overall composite (cascaded) amplifier.
- The gain of each stage can be calculated separately, then the total gain is found by multiplying the resulting gains

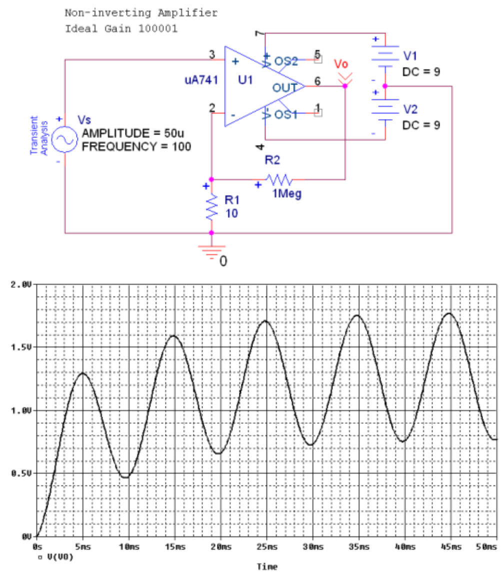


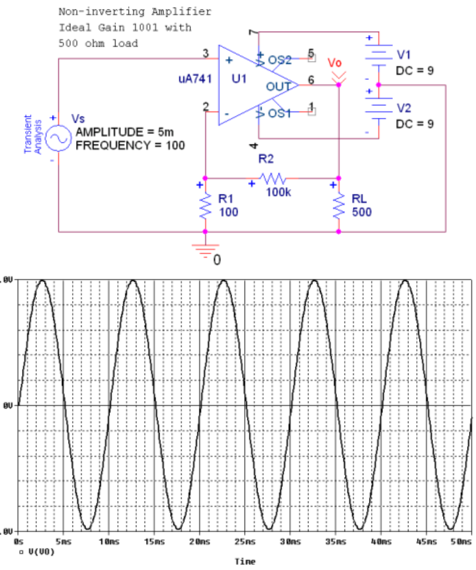
$$A = A_1 A_2 A_3$$

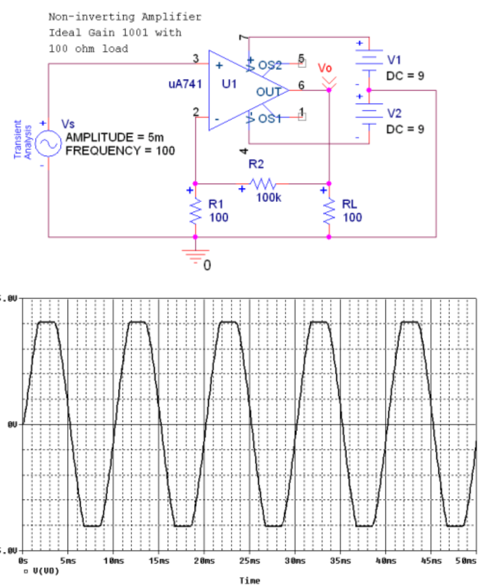
where $A_1 = 1 + R_f/R_1$, $A_2 = -R_f/R_2$, and $A_3 = -R_f/R_3$.

Example PSpice Simulations of Non-inverting Amplifier Circuits

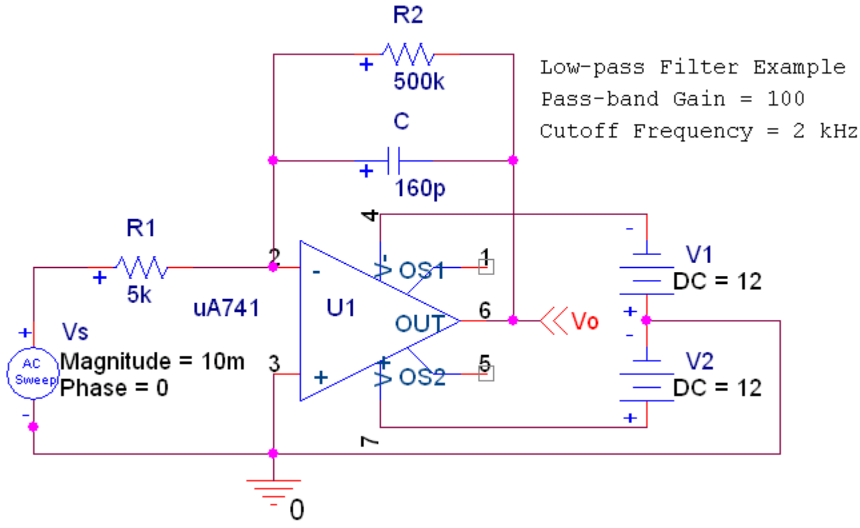

Non-inverting Amplifier
Ideal Gain 1001
uA741
U1
OUT
OS2
OS1
Vo
V1
DC = 9
V2
DC = 9
Transient Analysis
Vs
AMPLITUDE = 5m
FREQUENCY = 100
R2
100k
R1
100
0
5.0V
0V
-5.0V
0s
5ms
10ms
15ms
20ms
25ms
30ms
35ms
40ms
45ms
50ms
V(VO)
Time


Non-inverting Amplifier
Ideal Gain 100001
uA741
U1
OUT
OS2
OS1
Vo
V1
DC = 9
V2
DC = 9
Transient Analysis
Vs
AMPLITUDE = 50u
FREQUENCY = 100
R2
1Meg
R1
10
0
2.0V
1.5V
1.0V
0.5V
0V
0s
5ms
10ms
15ms
20ms
25ms
30ms
35ms
40ms
45ms
50ms
V(VO)
Time

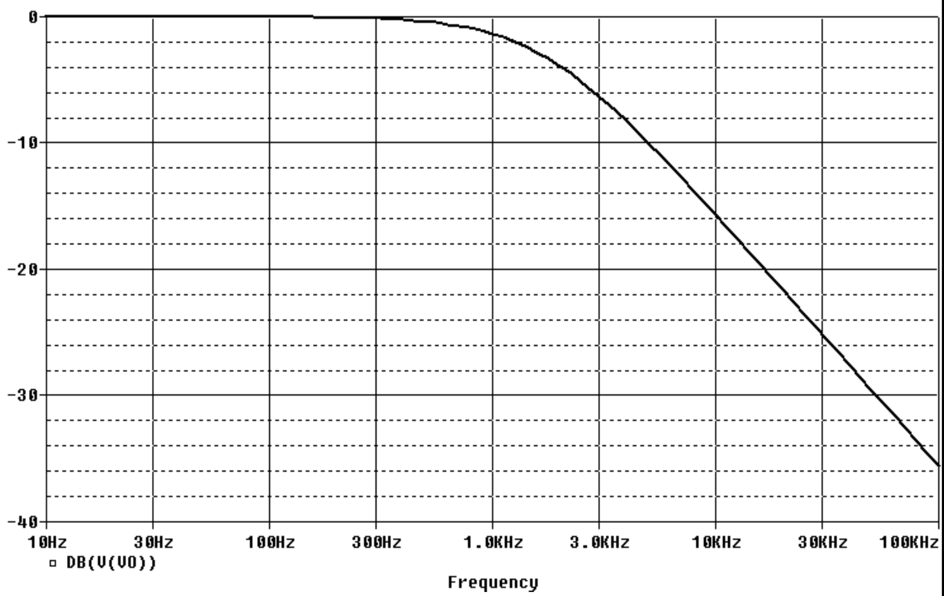

Non-inverting Amplifier
Ideal Gain 1001 with
500 ohm load
uA741
U1
OUT
OS2
OS1
Vo
V1
DC = 9
V2
DC = 9
Vs
AMPLITUDE = 5m
FREQUENCY = 100
Transient Analysis
R2
100k
R1
100
RL
500
0
5.0V
0V
-5.0V
0s
5ms
10ms
15ms
20ms
25ms
30ms
35ms
40ms
45ms
50ms
V(VO)
Time


Non-inverting Amplifier
Ideal Gain 1001 with
100 ohm load
uA741
U1
OUT
OS2
OS1
Vo
V1
DC = 9
V2
DC = 9
Vs
AMPLITUDE = 5m
FREQUENCY = 100
Transient Analysis
R2
100k
R1
100
RL
100
0
5.0V
0V
-5.0V
0s
5ms
10ms
15ms
20ms
25ms
30ms
35ms
40ms
45ms
50ms
V(VO)
Time

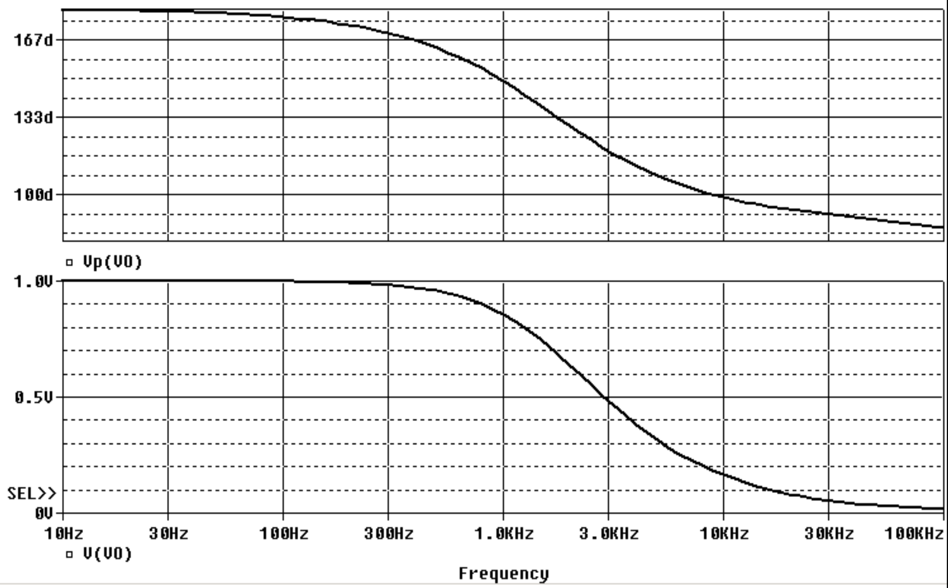
Low-pass Filter Example PSpice Simulation



Output Voltage Amplitude in dB



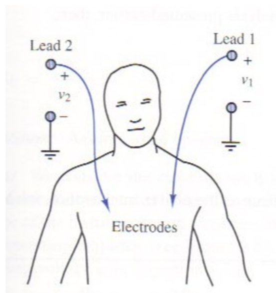
Output Voltage Amplitude in Volts (V) and Phase in Degrees (d)

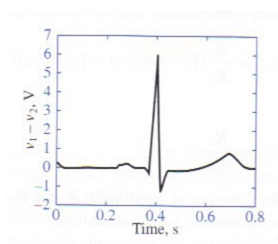


Following Material is for Reference Only

Applications of Op-Amps

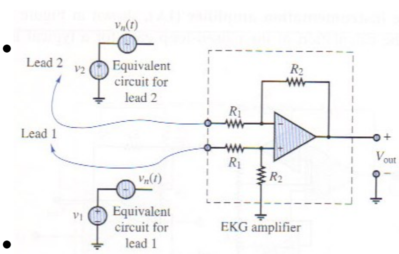
- Electrocardiogram (EKG) Amplification
  - Need to measure difference in voltage from lead 1 and lead 2
  - 60 Hz interference from electrical equipment

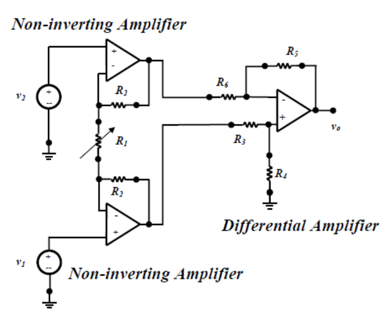




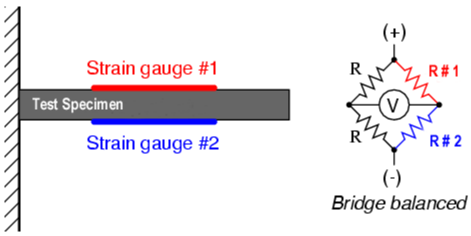
Applications of Op-Amps

- Simple EKG circuit
  - Uses differential amplifier to cancel common mode signal and amplify differential mode signal
- Realistic EKG circuit
  - Uses two non-inverting amplifiers to first amplify voltage from each lead, followed by differential amplifier
  - Forms an "instrumentation amplifier"

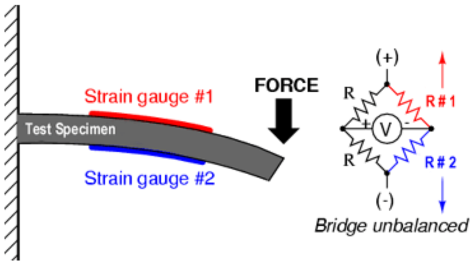




Strain Gauge



Use a Wheatstone bridge to determine the strain of an element by measuring the change in resistance of a strain gauge



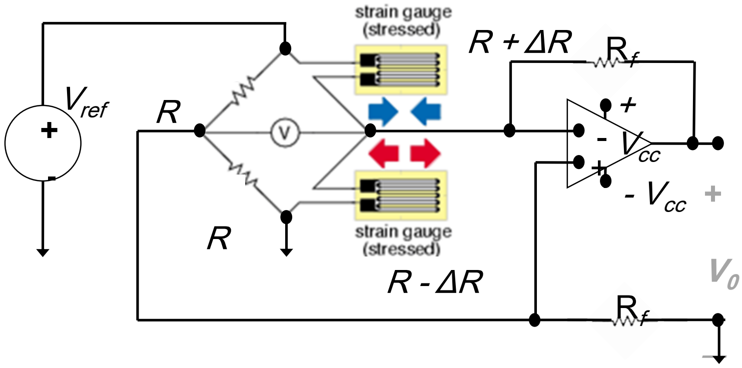
(No strain) Balanced Bridge
R #1 = R #2

(Strain) Unbalanced Bridge
R #1 ≠ R #2

Strain Gauge

Half-Bridge Arrangement

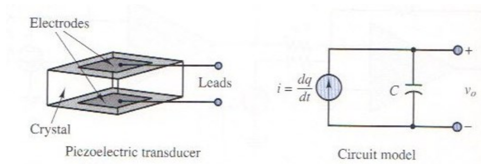
Op amp used to amplify output from strain gauge



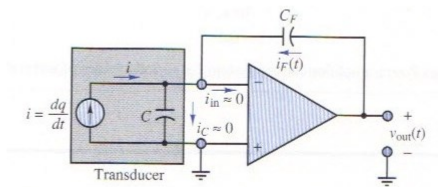
Using KCL at the inverting and non-inverting terminals of the op amp we find that → $\varepsilon \sim V_o = 2\Delta R(R_f/R^2)$

Applications of Op-Amps

- Piezoelectric Transducer
  - Used to measure force, pressure, acceleration
  - Piezoelectric crystal generates an electric charge in response to deformation

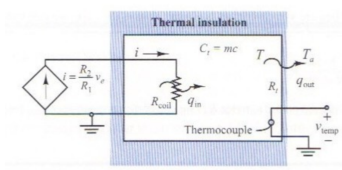


- Use Charge Amplifier
  - Just an integrator op-amp circuit



Applications of Op-Amps

- Example of PI Control: Temperature Control
- Thermal System we wish to automatically control the temperature of:



- Block Diagram of Control System: